NTIN071 A&G: Tutorial 1 – Deterministic finite automaton, recognized language, regular languages

Teaching goals: The student is able to

- use basic terminology and notation from formal languages and automata theory
- explain the formal definition of a DFA, and the language it recognizes
- describe a language recognized by a given DFA, in set notation
- construct (and describe formally) a DFA recognizing a given language
- prove the closure of regular languages under basic set operations

In-class problems (think-pair-share)

Problem 1 (Constructing a DFA for a given language)**.** Construct a DFA recognizing the given language.

(a) $L = \{w \in \{a,b\}^* \mid |w|_a \text{ is not divisible by } 3\}$

(b) $L = \{w \in \{a,b\}^* \mid 2 \text{ divides } |w|_a \text{ or } 3 \text{ divides } |w|_b\}$

(c) $L = \{w \in \{a,b\}^* \mid 2 \text{ divides } |w|_a \text{ and } 3 \text{ divides } |w|_b\}$

(d) $L = \{w \in \{0,1\}^* \mid w \text{ is a binary encoding of a nonnegative integer divisible by } 3\}$

Problem 2 (DFA given by a table)**.** Draw a state diagram and describe the recognized language in set notation.

(a)

	0	1
$\to$ p	q	p
$*$ q	r	q
$*$ r	p	r

(b)

	0	1
$\to$ p	p	q
$*$ q	r	q
$*$ r	p	q

Problem 3 (Describing a language and constructing a DFA for a given property)**.** Construct a DFA accepting exactly those words over the alphabet $\Sigma = \{a,b\}$ that satisfy the given property. Describe the language in set notation.

(a) starts 'abba' (b) ends 'abba' (c) contains 'abba' or 'bab' as a subword

Problem 4 (Regular languages and set operations)**.** Let L, L' be regular languages over the same alphabet. Show that the following is true:

(a) $\Sigma^* \setminus L$ is a regular language

(b) $L \cup L'$ is a regular language

(c) $L \cap L'$ is a regular language

Extra Practice and Thinking

Problem 5. Construct a DFA recognizing the given language.

(a) $L = \{w \in \{a, b\}^* \mid |w|_a \text{ is even}\}$

(b) $L = \{w \in \{a, b\}^* \mid |w|_b \text{ is divisible by } 3\}$

(c) $L = \{w \in \{a, b\}^* \mid 2 \text{ or } 3 \text{ divides } |w|_a\}$

(d) $L = \{w \in \{a, b\}^* \mid 2 \text{ and } 3 \text{ divides } |w|_a\}$

(e) $L = \{w \in \{0, 1\}^* \mid w \text{ is a binary encoding of a nonnegative integer divisible by } 5\}$

Problem 6. Draw a state diagram and describe the recognized language in set notation.

(a)

	0	1
→ ∗ p	q	p
q	r	q
r	p	r

(b)

	0	1
→ p	p	q
q	p	r
∗ r	p	r

Problem 7. Construct a DFA recognizing the language of all words over $\Sigma = \{a, b\}$ satisfying the property:

(a) has at least 2 letters and the first letter is the same as the last letter

(b) has at least 2 letters and the first two letters are the same as the last two letters

Problem 8. For $w \in \{0, 1\}^*$ let $h(w)$ be the remainder modulo 2 of the non-negative integer whose binary representation is w, allowing leading zeros. More precisely, if $w = w_1 w_2 \cdots w_{|w|}$ where $w_i \in \{0, 1\}$, then $h(w) = \left(\sum_{i=1}^{|w|} w_i \, 2^{|w|-i}\right) \bmod 2$. For the empty word we have $h(\varepsilon) = 0$.

Construct a DFA accepting the language $L = \{\, u\#v \mid u, v \in \{0, 1\}^* \text{ and } h(u) = h(v) \,\}$ (over the alphabet $\Sigma = \{0, 1, \#\}$). Draw its state diagram and describe the automaton formally as well.

Problem 9. What if L, L' are regular languages over different (but not necessarily disjoint) alphabets? Are the languages $\Sigma^* \setminus L$, $L \cup L'$, $L \cap L'$ necessarily regular?

Problem 10. Would you be able to show that L^R (i.e., words from L written in reverse) is regular whenever L is regular?