NTIN071 A&G: Tutorial 3 – Testing properties, nondeterminism, subset construction, closure under set operations

Teaching goals: The student is able to

- design efficient algorithms to test basic properties of automata
- define NFA, ϵ-NFA, and the languages they recognize
- apply the subset construction to convert an NFA or ϵ-NFA to a DFA
- define automata homomorphism and isomorphism
- check for existence of homomorphisms and isomorphisms between automata
- decide if regular languages are closed under various set operations

In-class problems

Problem 1 (Testing properties)**.** Consider finite automata A, B. Design an algorithm that decides the following properties. (Can you estimate the time complexity?)

(a) $L(A) = \emptyset$, (b) $L(A) = L(B)$, (c) $L(A) \subseteq L(B)$, (d) $L(A)$ is finite.

Problem 2 (Subset construction)**.** Given a nondeterministic finite automaton with ϵ-transitions, construct an equivalent reduced DFA.

	0	1	ϵ
$\rightarrow A$	$\{E\}$	$\{B\}$	$\emptyset$
B	$\emptyset$	$\{C\}$	$\{D\}$
$\rightarrow C$	$\emptyset$	$\{D\}$	$\emptyset$
$*D$	$\emptyset$	$\emptyset$	$\emptyset$
E	$\{F\}$	$\emptyset$	$\{B, C\}$
F	$\{D\}$	$\emptyset$	$\emptyset$

Problem 3 (Automata homomorphism)**.** Find DFA A, B such that:

(a) Both are reduced and they are not isomorphic.
(b) A is homomorphic onto B but they are not isomorphic.
(c) They are equivalent but not isomorphic.
(d) They are both homomorphic onto C but not isomorphic with it. Moreover, A is not homomorphic onto B and B is not homomorphic onto A.

$$C = (\{p, q\}, \{0, 1\}, \{((p, 0), q), ((p, 1), p), ((q, 0), p), ((q, 1), q)\}, p, \{q\})$$

Problem 4 (Closure under set difference)**.** Given DFAs A, B, construct an automaton recognizing the language $L(A) - L(B)$. (Give a formal description of the construction.)

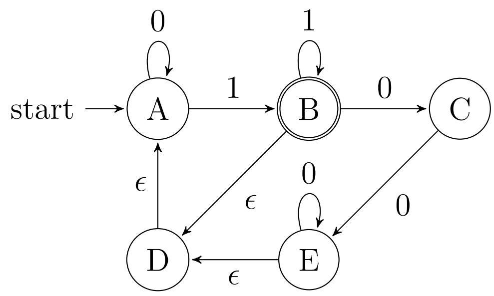
Extra Practice and Thinking

Problem 5 (Subset construction)**.** Construct an equivalent reduced DFA.

(a)

	0	1	ϵ
$*A$	$\{A, C\}$	$\{B\}$	$\emptyset$
B	$\{B, D\}$	$\emptyset$	$\emptyset$
$*C$	$\{E\}$	$\{D\}$	$\emptyset$
D	$\{A\}$	$\{C, D\}$	$\emptyset$
$\to *E$	$\emptyset$	$\emptyset$	$\{A, C\}$

(b)



Problem 6 (Regular?)**.** Let L be a language over the alphabet $\{a, b\}$ consisting of words that do not contain three consecutive occurrences of the same letter. Decide if L is regular. If it is, find a reduced DFA accepting L.

Problem 7 (Subset construction reduced)**.** Is the result of the subset construction (where we only generate reachable states) always a reduced DFA? Prove or disprove.

Problem 8 (Switch final and nonfinal states)**.** If we switch accepting and nonaccepting states in a given NFA, will the language recognized by the resulting automaton be the complement of the language recognized by the original NFA? Justify your answer.