# NTIN071 A&G: Tutorial 4 – Closure under string operations

**Teaching goals:** The student is able to

- formally describe a construction of an automaton based on other automata
- decide whether regular languages are closed under various string operations, including more complex ones, and prove or disprove it

## In-class problems

**Problem 1** (Closure under string operations)**.** Given a DFA $A$, construct an automaton recognizing the given language. (Give a formal description of the automaton.)

(a) $L(A).L(A)$

(b) $L(A)^+$

(c) $L(A)^*$

(d) $L(A)^R$

| | a | b |
|---|---|---|
| $\rightarrow 0$ | 1 | 2 |
| $*\ 1$ | 3 | 0 |
| 2 | 4 | 5 |
| 3 | 0 | 2 |
| 4 | 2 | 5 |
| 5 | 0 | 3 |

| | a | b |
|---|---|---|
| $\rightarrow 0$ | 0 | 5 |
| $*\ 1$ | 1 | 3 |
| 2 | 2 | 5 |
| 3 | 3 | 2 |
| $*\ 4$ | 6 | 1 |
| 5 | 5 | 1 |
| $*\ 6$ | 4 | 2 |

**Problem 2** (Delete)**.** Let $L$ be a regular language over the alphabet $\Sigma = \{a, b\}$. Describe the following languages in set notation. Decide if they are (necessarily) also regular, prove or disprove. The language of all words obtained from words of the language $L$ by…

(a) …deleting all occurrences of the letter $a$.

(b) …deleting the initial letter and writing this letter at the end of the word.

(c) …deleting the longest contiguous sequence of $a$'s from the beginning of the word.

## Extra Practice and Thinking

**Problem 3** (Prefixes)**.** Are regular languages closed under the following operations? Prove or disprove. (In the following, $L$ is a regular language over an alphabet $\Sigma$.)

(a) $\text{init}(L) = \{w \in \Sigma^* \mid \text{there is } u \in \Sigma^* \text{ such that } wu \in L\}$

(b) $\min(L) = \{w \in L \mid \text{there is no } u \in L, v \in \Sigma^+ \text{ such that } w = uv\}$

(c) $\max(L) = \{w \in L \mid \text{there is no } u \in \Sigma^+ \text{ such that } wu \in L\}$

**Problem 4** (Shift)**.** Given a regular language $L$ over an alphabet $\Sigma$, define the language $L'$ as follows. Is the language $L'$ necessarily regular?

$$L' = \{uv \mid u, v \in \Sigma^*, vu \in L\}$$

**Problem 5** (Cut)**.** Consider two regular languages $L$ and $M$ over an alphabet $\Sigma$, and define the language $K$ as follows. Is the language $K$ necessarily regular?

$$K = \{uw \mid u, w \in \Sigma^*, (\exists v \in M)\, uvw \in L\}$$

**Problem 6** (Iterations of unary languages)**.** Show that for any language $L$ over the alphabet $\Sigma = \{a\}$, the language $L^*$ is regular.