

Teaching goals: The student is able to

- decide whether context-free languages (CFL) are closed under various operations
- give a formal definition of a deterministic pushdown automaton (DPDA)
- explain the difference between accepting by final state and by empty stack for DPDA, i.e., the classes of languages L_{DPDA} and N_{DPDA}
- explain and illustrate the relationship between L_{DPDA} and N_{DPDA} with other classes
- decide whether L_{DPDA} and N_{DPDA} are closed under various operations

IN-CLASS PROBLEMS

Problem 1 (Non-palindromes). Consider the language $L = \{w \in \{a, b\}^* \mid w = w^R\}$.

- (a) Is L context-free?
- (b) Is its complement \bar{L} context-free?

Problem 2 (Closure Properties). Are (I) CFL, (II) L_{DPDA} , (III) N_{DPDA} closed under the following operations? Prove or disprove.

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|------------------|--|
| (a) Intersection | (c) Intersection with a regular language |
| (b) Complement | (d) Homomorphism |

Problem 3. Show that:

- (a) $\{a^n b^{n+1} \mid n \geq 0\} \in N_{DPDA}$
- (b) $\{a^n b^{n+k} \mid n \geq 0, k \in \{0, 1\}\} \in L_{DPDA} \setminus N_{DPDA}$
- (c) $\{a^n b^{n^2} \mid n \geq 0\} \in L_{DPDA}$
- (d) $\{a^n b^{n^k} \mid n \geq 0, k \in \{1, 2\}\} \in CFL \setminus L_{DPDA}$

EXTRA PRACTICE AND THINKING

Problem 4 (Closure Properties). Are (I) CFL, (II) L_{DPDA} , (III) N_{DPDA} closed under the following operations? Prove or disprove.

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| (a) Union | (e) Substitution of a regular language |
| (b) Concatenation | (f) Inverse homomorphism |
| (c) Iteration | (g) Right/left derivation |
| (d) Reversal | (h) Union with a regular language |